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Microcredit and Price Competition: standardize to differentiate

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Abstract

Microfinance institutions, despite the presence of competition and informational asymmetries, typically offer a limited variety of contracts. Assuming price competition, we propose a simple theoretical explanation for this behavior and study its consequences in terms of strategic interaction and borrower welfare. We model an oligopolistic market in which Microfinance Institutions design their contracts and choose how many of them to offer. We find that when offering a menu is costly, MFIs always offer a single contract. Despite that, there exist equilibria in which MFIs coordinate and offer screening contracts, allowing them to extract a large fraction of the borrower welfare. We discuss the policy implications of our model in terms of price caps, market entry and outreach measurement.

Keywords: Microfinance, Competition, Altruism, Contract Menus, Credit Rationing
JEL Classification: G21, L13, L31, O16

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1. Introduction

The microfinance industry is a variegated world. There are many different approaches to lending money to the poor: some lenders are market-oriented, some are motivated by principles of solidarity and assistance, and others try to mix the advantages of both approaches. Yet few key ingredients of the microfinance mechanism seem to be recurrent both in practitioners' and academic discussions. One of them is the role of the interest rate; another is the type and variety of contracts offered. We provide a simple model to discuss the importance of these elements and the way they interact with each other.

Economic theory suggests that the best way to cope with informational asymmetries is to offer a menu of contracts. Yet, despite operating in an environment characterized by moral hazard and adverse selection, most Microfinance Institutions (MFIs) offer a limited number of contract types (Rahman, 2007, Banerjee and Duflo, 2011, Dehejia et al., 2012). This behavior is usually justified by the argument that, in order to reduce operating costs and hire less specialized personnel, microfinance products must be as streamlined and standardized as possible. Some practitioners also argue that offering different conditions to different clients in the same area can cause confusion and discontent between clients. In some cases, additional financial products are proposed to borrowers as a test, and are therefore offered on limited scale. Moreover, contracts are very often related to a particular lending mechanism (for instance, group versus individual lending) and the interest rate is tailored to such mechanisms.

Undoubtedly, MFIs are now making an effort to offer a wider range of products. Some institutions offer both individual and group lending, others design financial products that better fit the needs of a specific business sectors. For instance, loan contracts designed for farmers - whose revenues are strongly seasonal - have a different frequency of repayments compared to contracts designed for petty traders. This type of differentiation is based on *ex-ante* verifiable characteristics of the borrowers as, for instances, their profession. But within a particular group of borrowers, there can still be relevant issues related to asymmetric information.

We present a simple model in which competing MFIs can decide the type and the number of contracts to offer. We consider a sequential framework with asymmetric information in which two firms (Incumbent and Entrant) compete for two types of borrowers (Safe and Risky).[1] MFIs can screen borrowers using the interest rate and the loan size as strategic variables (Stiglitz and Weiss, 1981). We first assume that both firms are profit maximizing. We then consider the case in which the Incumbent is altruistic, i.e. maximizes the borrower welfare under a non-bankruptcy constraint.

We assume that MFIs face a market characterized by adverse selection and, due to environmental, technological and labour market features, offering a variety of contracts (a menu) is more costly than offering a standardized one. We show that the cost of the menu influences in a non trivial way the type and the number of contracts offered by competing MFIs. In particular, MFIs never offer more than one contract when the cost of a menu is positive. When the cost is low, only a zero profit equilibrium in which MFIs offer the same contract is possible. However, when the cost is more substantial, MFIs differentiate their contracts in order to screen borrowers. Safe borrowers are rationed (i.e. their loan size is smaller than what they demand), whereas Risky borrowers enjoy a positive rent. Our model propose a mechanism to explain how, in a competitive microcredit market, the large costs faced by lenders are reflected in the interest rates. MFIs, in order to reduce costs, coordinate in a screening strategy that mimics the behavior of a monopolist. Each of them offers a single contract that, paired with the one offered by the competitor, makes screening possible. This generates a perverse effect in terms of borrower welfare, since MFIs are able to extract a large share of the borrower welfare. When the cost of the menu is low, screening is not sustainable in equilibrium, and price competition drives the interest rate down to a pooling, zero profit equilibrium.

It has been repeatedly argued that the demand for microcredit is relatively inelastic to the interest rate. There is a widespread belief that poor clients are so starved for credit that they are willing to pay almost any interest (CGAP, 1996). Evidence for this is scattered.

[1] The sequential structure of the game is very helpful in easing exposition, but is not essential, since all the results are also valid in a simultaneous setting. A formal proof is available upon request.

Porteus (2006) reports that interviews conducted with MFI managers in Bolivia, Uganda and Bangladesh seem to confirm that clients consider the interest as less important relative to other factors such as loan size, non-financial services and lending methodology. However, the responsiveness to interest rates is related to a number of underlying characteristics of the market. The low level of financial literacy of microfinance clients, for instance, or the level of competition can play a fundamental role (Karlan and Morduch, 2010).

Yet there is evidence that interest rates have begun to matter to microborrowers. According to a survey conducted by MicroSave in Uganda, "clients have repeatedly cited interest rates as one of the top determinants of their choice of the financial service provider from whom they borrow." (Wright and Rippey, 2003). Dehejia et al. (2012), using a quasi-experimental setting in Bangladesh, estimate the interest elasticity and find that an increase in price leads to substantial changes in the demand. Similar, albeit more nuanced, results are presented by Karlan and Zinman (2008). Using randomized controlled trials in South Africa, they find a modestly negative elasticity, but their results are based on consumer loans that differ in some features from standard microfinance loans. Karlan et al. (2009), using a similar methodology, find Compartamos's clients in Mexico to be strongly sensitive to interest rates. Besides this evidence, the increasing spread of microfinance is very likely to induce an improvement of the borrowers' financial literacy. As a consequence, the interest rate should be considered more and more as a relevant decision variable in competitive environments (Porteus, 2006). Building on these observations, our model, discusses the effects of competition in a market in which microborrowers consider the interest rate as a key variable to select their favorite loan.

Our simple model is also a useful tool through which approach three issues widely debated in the literature. First, we analyze the consequences of setting a price cap. Our model shows that, although beneficial for borrowers, price caps can have unintended consequences on MFIs' profitability. Second, we explore the effects of altruism on the entry of competitors. Surprisingly, our model predicts that the presence of an altruistic MFI can foster entry by facilitating the screening of borrowers. Last, we discuss the implications of our results for the empirical literature using the size of the loan as a proxy for the depth of outreach (among others, Cull et al., 2009b, Cull et al., 2007, Mersland and Øystein Strøm, 2010). We suggest that some caution is needed when using this measure in a market characterized by asymmetric information, since the size of the loan can be used as a screening device and therefore does not necessarily reflect the MFIs' mission drift.

Other papers have examined the issue of increasing competition in microcredit markets, but very few theoretical models are available. The most notable exception is McIntosh and Wydick (2005), who present a model in which MFIs maximize the number of borrowers served, and cross-subsidize the non-profitable borrowers using the profits earned by serving the profitable ones. They show that as competition increases and profits shrink poor borrowers are excluded from credit. Their result is based on the assumptions that poor borrowers are less profitable than richer ones, and that MFIs can offer a different contract for each borrower. We will assume instead that all borrowers give *ex-ante* the same expected profit (although they differ in their level of risk), and that contracts are standardized at the firm level. A more recent model is presented in de Quidt et al. (2012). They consider explicitly the problem of strategic default, and relate it to the lending mechanism (group vs. individual lending), the market structure (monopoly vs. free entry) and the behavior of lender (for-profit or benevolent). In their setup, competition leads to an allocation similar to the one induced by a benevolent monopolist.

Other papers take an empirical approach. McIntosh et al. (2005), using data from Uganda, show that the entry of new MFIs does not influence the drop-out rate, but does deteriorate the repayment performance. They also study the location decision of MFIs, and find a tendency towards the creation of clusters of institutions. Our model provides a possible explanation for this phenomenon. Their estimations show no evidence of changes to the loan size, which is in contrast with our findings. But, as discussed by the authors, this is likely due to the fact that the Ugandan microcredit market is far from being saturated. Navajas et al. (2003) describe the Bolivian microcredit market and its evolution from monopoly to duopolistic competition. They explain how the two main institutions in the market (Bancosol and Caja Los Andes) specialized in different market niches, offering different contracts to target different types of borrowers. This pattern seems to be common in microcredit markets. Cull et al. (2009a) explore the effects of competition between banks and microbanks in terms of profitability and the outreach of MFIs. They show that bank penetration induces MFIs to offer smaller loans. Our model suggests that a very similar mechanism is also at work when MFIs compete with each other. But in contrast to their approach, we show that reduction of the loan size can arise also when clients differ only in term of riskiness (and not in terms of wealth).[2] Baquero et al. (2012) measure the effects of competition on interest rates and financial performance. They find that higher concentration is associated with higher interest rates, but that nonprofit institutions seem to respond less to the competitive pressure. Floro and Ray (1997) describe the interaction of formal and informal lenders in the Philippines, and argue that increasing competition might not be beneficial.

There are many studies analyzing the effect of competition between banks. The work of Villas-Boas and Schmidt-Mohr (1999) is the closest to our approach. They study an oligopolistic credit market in which firms differentiate their contracts. They show that more competition leads to more screening, but different to our approach, they assume that banks identify the best clients through the collateral they are willing to post. Our results are similar, but the screening mechanism we propose differs so as to better describe the microfinance markets. Other papers study the incentives to share borrowers' information with competitors (Van Tassel, 2011, Bouckaert and Degryse, 2006), or the impact on relationship lending (Petersan and Rajan, 1995).

The paper is organized as follows: In Section 2 we introduce the basic model and describe the Entrant's reaction function. We then analyze the behavior of an Incumbent that

[2]One may, of course, argue that the level of riskiness is related to the level of poverty. But, if anything, the microfinance experience shows that, when served with the right mechanism, poor people repay their loans as much as, or more than, their wealthier counterparts.

maximizes profits. We show how and when differentiation takes place. Building on these results, in Section 3, we analyze the choice of the optimal menu of contracts. In Section 4 we modify the basic setup to model the behavior of an altruistic Incumbent and highlight the role of capacity constraints. In Section 5 we discuss the policy implications of our model in terms of price caps, market entry and outreach measurement. In Section 6 we conclude.

2. The Basic Model

Consider a microcredit market served by a single MFI (the Incumbent), and suppose that a second one (the Entrant) is considering entering the market. There is a unit measure of borrowers demanding a one unit loan to finance a new business. There is a fraction β of safe borrowers characterized by a return R_s and a probability of success p_s, and a fraction $1-\beta$ of risky borrowers with return R_r and probability of success p_r. The identity of the borrowers is private information. We assume that $p_i R_i = m > 1$ and that $p_s > p_r$. Hence, $R_s < R_r$. This ensures that both types have the same expected return. These assumptions imply that if MFIs were informed about the riskiness of their clients, they would be *ex-ante* indifferent between serving either type of borrowers. However, since MFIs cannot distinguish borrowers, they forgo some profits (or make losses) if they serve Risky borrowers with a contract designed for Safe borrowers.

Contracts are defined as a pair $C = (x, D)$, in which MFIs specify the repayment $D \geq 1$, inclusive of principal and interests, and the loan size $x \in [0, 1]$.[3] We denote by $C^I = (x^I, D^I)$ the contract offered by the Incumbent, and by $C^E = (x^E, D^E)$ the contract offered by the Entrant. The borrowers' type is private information. As a tie-breaking rule, we assume that even when the contract leaves the borrowers with no rent, they still prefer borrowing to not borrowing. We assume that MFIs can offer a single contract type incurring a cost normalized to zero. Conversely, if they opt for a menu of contract, they incur a fixed cost $k > 0$.

The timing is the following: at time $t = 1$ the Incumbent sets her menu of contract(s). The Entrant observes the market and the Incumbent's strategy and at time $t = 2$ she decides whether to enter the market with her own menu. At time $t = 3$, borrowers observe both menus and choose their favorite.

We solve the model considering first the Entrant's optimal reaction for any given choice by the Incumbent, and then proceed through backward induction to specify the optimal choice by the Incumbent.

Note that any contract found acceptable by the Safe borrowers attracts also the Risky ones since $R_s < R_r$. Thus, if only one MFI were in the market, she could only decide on whether to serve the risky or both types. When two MFIs compete, instead, there is another possibility: an MFI can choose to serve only the Safe borrowers, since the competitor can help to screen out one type from the other by offering an incentive compatible contract.

[3] Alternatively, x_i could be interpreted as random rationing, i.e. as the probability of a borrower being served.

When comparing the contracts offered by the Incumbent and the Entrant, borrowers are solely concerned with the monetary outcome. The demand faced by each MFI depends on C^I and C^E. We define the demand function as $B^i(C^I, C^E)$, with $i = I, E$ that assigns to each combination of contracts the mass of borrowers preferring MFI i.

For the ease of exposition, we start by describing the best strategies of both Incumbent and Entrant under the working assumption that k is so large that no MFI is willing to offer more than one contract. We relax this assumption in Section 3, where we characterize the full equilibrium.

2.1. The Entrant Strategy

At time $t = 2$ the Entrant chooses her contract upon the observation of the Incumbent's choice. She has two possibilities: (i) Offer a contract that attracts only borrowers of a specific type; (ii) Offer a contract that attract both types. The first option is only feasible if the Incumbent sets a contract that allows screening. Let $P^i(C^I, C^E)$ be the function assigning to each combination of contracts the average probability of repayment of the borrowers served by MFI i. It takes value p_r, p_s or $p_b := \beta p_s + (1-\beta)p_r$ when MFI i serves respectively the Risky, the Safe or Both types of borrowers. The Entrant faces the following maximization problem:

$$\max_{x^E, D^E} \Pi^E = x^E B^E(C^I, C^E)\Big[P^E(C^I, C^E)D^E - 1\Big]$$

The Entrant's strategy set is given by the set of all possible contracts (x, D) such that $x \in [0, 1]$ and $D \geq 1$. But the strategy set can be divided in three subsets, each of them identifying a possible *intention*: serving the Risky, the Safe or Both borrower types. In other words, the choice of a contract determines whether there will be direct competition (both MFIs targeting the same pool of borrowers) or full separation (each MFI specializing in a particular group).

MFIs can ration borrowers in order to make screening possible. By properly adjusting the value of x, they can reduce the expected profitability of the contract designed for the Safe borrowers. At the same time, the Risky borrowers can be given an informational rent. In what follows, we prove the existence of equilibria in which MFIs find it profitable to design screening contracts in order to make differentiation possible.

Screening Strategies. Since the Entrant's contract is chosen after the observation of the Incumbent's choice, the Incumbent can induce the Entrant to serve a particular market niche and engage in a screening strategy. She can do so by offering a contract that makes it optimal for the Entrant to target only one type of borrower. The aim of this section is to check under which conditions two competing institutions can set contracts such that borrowers reveal their identity by self-selecting the loan they prefer. Of course, Risky borrowers generally have an interest in 'pretending' they are safe. In order to avoid this, the contract designed to serve Safe borrowers, can be made less appealing by reducing the amount lent ($x_s < 1$). At the same time, the contract designed for the Risky borrowers can be made more interesting by leaving a positive rent (the informational rent). We explain the mechanism in the next two lemmas.

Lemma 1. *If the Incumbent chooses a contract such that $D^I \leq R_s$ and $x^I \leq \hat{x}_s^I(D^I) \in (0,1)$ then the Entrant's optimal reaction is to offer a contract $C^{E*} = \Big(1; R_r - \frac{x^I}{x^E}(R_r - D^I)\Big)$, so that screening takes place with the Incumbent serving the Safe borrowers and the Entrant serving the Risky ones.*

Proof. See Appendix □

The value of $x^I < \hat{x}_s^I(D^I)$ is defined in the Appendix. The intuition behind this result is standard: if the Incumbent wishes to serve only the Safe borrowers, she must ration them by setting $x^I \leq \hat{x}_s^I(D^I)$. Since in this basic model each MFI offers only one contract, the amount of rationing x^I must be set according to the Entrant's best outside option, which in this case, is to serve both types setting $D^E = D^I$. By doing so, the Entrant can earn:

$$\Pi_{Both}(D^I) := \beta(p_s D^I - 1) + (1-\beta)(p_r D^I - 1)$$

The Entrant's profit increases as x^I decreases. So x^I must be low enough to make screening possible.

To understand why, note that the level of rationing is inversely proportional to the informational rent: the higher the informational rent given to the Risky borrowers, the lower the level of rationing needed to induce self-selection of the contracts. But the Entrant's profit from serving only the Risky borrowers is lowered by the informational rent that her customers must be given. Thus, the more Safe borrowers are rationed, the higher the Entrant's profit. $\hat{x}_s^I(D^I)$ is calculated as to make the Entrant's profit equal to her outside options.

Note that we did not make any assumption about the profit the MFIs make if they serve their borrowers with the 'wrong' contract. In other words, $(p_r R_s - 1)$ might be negative, implying that if the MFIs do not screen their clients, the Safe borrowers subsidize the Risky ones. For this reason, Π_{Both} can be negative when β is small. In this case the Entrant's outside option is nil.

The Incumbent behaves the way explained above whenever serving the Safe market niche is her best strategy. When this is not the case, she can specialize in the Risky market niche, inducing the Entrant to target the Safe borrowers. In order to do so, she has to grant the Risky borrowers an adequate informational rent, allowing the Entrant to ration as little as possible the Safe borrowers. The mechanism is detailed in the next lemma.

Lemma 2. *(i) If the Incumbent offers a contract $(x^I, \tilde{D}_r^I(x^E))$ characterized by:*

$$R_s \leq \tilde{D}_r^I(x^E) := R_r - \frac{1}{x^I} x^E (R_r - D^I) \tag{1}$$

where $x^E \in [\tilde{x}_s, 1)$, then the Entrant's optimal reaction is to offer a contract characterized by $x^E = \tilde{x}_s$ and $D^E = R_s$, so that screening takes place with the Incumbent serving the Risky

borrowers and the Entrant serving the Safe ones.
(ii) If the Incumbent offers a contract $(1, D^I)$ *characterized by* $\max\{D^I_{min}, 1/p_r\} \leq D^I < R_s$, *then the Entrant's optimal reaction is to offer a contract characterized by* $x^E = 1 - \epsilon$ *and* $D^E = D^I - \epsilon$, *with* $\epsilon \in \mathbb{R}_+$ *arbitrarily small, so that screening takes place with the Incumbent serving the Risky borrowers and the Entrant serving the Safe ones*

Proof. See Appendix. □

The value of $\tilde{x}_s$ is defined in the Appendix. D^I_{min} denotes the minimum value of D^I making the Entrant indifferent between the screening profit and the relevant outside option, and $1/p_r$ is the value of D^I such that the Incumbent breaks even. The mechanism is the same as in the previous lemma. The Entrant has, in this case, two possible outside options: serving both types (earning Π_{Both}) or undercutting the Incumbent. The ranking between them is ambiguous: the first outside option is relevant when β is large. Note that the Incumbent could set $D^I < R_s$ and still be willing to serve the Risky borrowers. This case, described in point (ii), is relevant when the Incumbent is altruistic and maximizes the borrower welfare. (see Section 4).

2.2. The Incumbent Strategy: The Profit-Maximizing case

We now have all the elements required to analyze the Incumbent's optimal strategy. We start by assuming that the Incumbent MFI is *profit-maximizing*. Despite the presence of many socially-motivated institutions, some of the biggest and most influential MFIs do claim to be able to earn profits, and consider this ability the result of a careful, market oriented, management. According to a number of researchers and practitioners, commercializing microfinance is the best way to serve the poorest. It is argued that this approach provides better incentives for managers to be efficient and to attract investors (Rhyne, 1998; Prahalad, 2004; Christen and Drake, 2002).

Let $C^E(C^I)$ be the Entrant's reaction function to the Incumbent's strategy. The Incumbent faces this maximization problem:

$$\max_{x^I, D^I} \Pi^I = x^I B^I(C^I, C^E(C^I)) \left[P^I(C^I, C^E(C^I))D^I - 1\right]$$

The Incumbent, just like the Entrant, can choose whether to specialize in a particular market niche (Safe or Risky borrowers) or to target both types of borrowers. In the first case, she needs to induce the Entrant to offer an incentive compatible contract as showed in Lemmas 1 and 2. In what follows, we describe her optimal behavior for each possible case.

The Incumbent serves the Safe borrowers. If the Incumbent wants to attract only Safe borrowers, she needs to offer a contract satisfying the conditions in Lemma 1, thereby inducing the Entrant to target the Risky borrowers offering an incentive compatible contract. When the Incumbent is profit-maximizing, the Entrant's dominant outside option is to undercut the Incumbent's contract setting $x^E = 1$ and $D^E = D^I$. Such a contract attracts all borrowers and makes screening impossible. $\hat{x}^I_s(D^I)$, as defined in Lemma 1, is set to avoid the

Entrant playing this strategy. Since $\hat{x}^I_s(D^I)$ is increasing in D^I, the Incumbent sets D^I as large as possible, taking into account the constraint $D \leq R_s$. This leads to $D^I = R_s$. If the constraint in Lemma 1 is not binding, then the Incumbent can set any $x^I < 1$. Under these conditions $B^I(C^I, C^E) = \beta$, and the Incumbent's expected profit is:

$$\Pi^I_s = \beta \hat{x}_s(R_s)(m-1). \tag{2}$$

The Incumbent serves the Risky borrowers. If the Incumbent wants to serve the Risky borrowers, she has to induce the Entrant to target the Safe ones with an incentive compatible contract. From Lemma 2 we know that $\tilde{D}^I_r(\cdot)$ is increasing in x^I, so the Incumbent chooses $x^I = 1$, and $D^I = \tilde{D}^I_r(1)$. This gives her the expected profit:

$$\Pi^I_r = (1-\beta)(p_r \tilde{D}^I_r(1) - 1) \tag{3}$$

The Incumbent serves both types. In this case, the Incumbent needs to avoid the Entrant undercutting her contract. Thus, she sets D^I as low as possible, namely $D^I = 1/p_b$, with $p_b := (\beta p_s + (1-\beta)p_r)$, and breaks even.

In order to choose her optimal strategy, the Incumbent has then to compare equations (2) and (3). Under our assumptions, the model has a simple equilibrium. Define $\beta_0 := \frac{m(R_r - R_s)}{m(2R_r - R_s) - R_r}$. We then have:

Proposition 1 (Equilibrium Characterization)**.** *Consider the model with profit-maximizing MFIs:*
(i) If $\beta < \beta_0$, we have a separating (screening) equilibrium in which the Incumbent serves the Safe borrowers setting $C^{I*} = (\hat{x}^I_s(R_s); R_s)$ and the Entrant serves the Risky borrowers setting $C^{E*} = (1; R_r - \hat{x}^I_s(R_s)(R_r - R_s))$. Both MFIs make positive profits.
(ii) If $\beta > \beta_0$ or $p_r R_s > 1$, then we have a pooling equilibrium in which both MFIs set $C^* = (1, 1/p_b)$ and make no profits.

Proof. See 7. □

The mechanism driving this result can be better understood by looking at Figure 1. Π^I_s and Π^I_r (the continuous and the dashed line respectively) are both piecewise functions of β. In fact, depending on the value of β, the Entrant's best outside option to the screening strategy changes. The Incumbent needs to adapt her contract (and therefore profit) to the prevailing outside option as explained in Lemmas 1 and 2. Proposition 1 shows that a screening equilibrium is possible only when the Incumbent serves the Safe borrowers.

To understand why, imagine first that, in equilibrium, the Incumbent targets the Safe borrowers and the Entrant the Risky ones. As β increases, the profit from serving the Safe borrowers increases. On the other hand, also the outside option of the Entrant (serving both types) increases since Π_{Both} is increasing in β. So the level of rationing must increase. As β grows, the second effect dominates, so that the curve Π^I_s decreases (and eventually becomes negative).

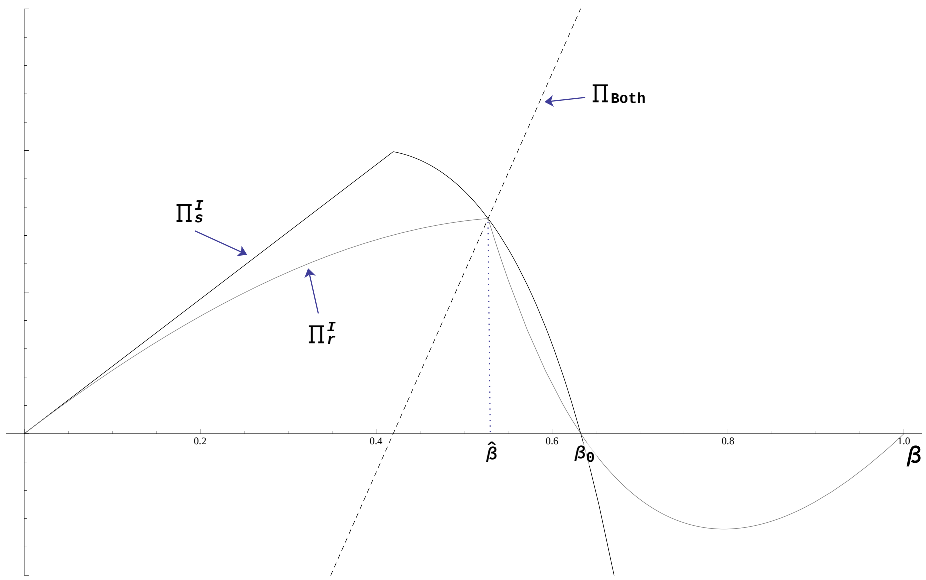

Figure 1: Profit Maximizing Model: Incumbent's profit as a function of β.

Consider now a situation with inverted roles. Consider first the interval $[0, \hat{\beta}]$, where the Entrant's prevailing outside option is to serve the Risky borrowers, undercutting the Incumbent. As β increases, the outside option becomes less attractive compared to the screening strategy, and the profit of the Incumbent also diminishes. But the first effect prevails, so the Incumbent profits are increasing and concave. In the interval $[\hat{\beta}, \beta_0]$, the relevant Entrant's outside option is to serve both types setting $D^E = R_s$. Here, as β increases the Entrant's outside option increases (and so her screening profit) and, at the same time, the Incumbent's profit decreases. Both effects go in the same direction, causing a sharp decrease of Π_r^I, that always lies below Π_s^I.

Note that our results are robust to the introduction of a capacity constraint for MFIs.

3. The optimal number of contracts

Suppose now that k is not as prohibitively high as in the previous section, so MFIs can consider offering a variety of contracts. MFIs trade off the cost of a menu with the extra profit that they might earn by screening borrowers according to their type. For simplicity, we assume that if both MFIs offer the same contract for the same type, then each MFI serves half of the pool of clients ($\beta/2$ or $(1-\beta)/2$). Moreover, the extra cost of the menu is equally distributed over all borrowers.[4] We show that, under our assumptions, offering a menu of contracts cannot be an equilibrium. This analysis also allows us to show formally that there

[4] In principle, MFIs can decide to cross-subsidize one type of borrower, charging the whole cost of the menu on the other type. Our assumption makes the analysis simpler, without affecting the results.

exists a $\overline{k}$ such that for any $k \geq \overline{k}$, the market equilibrium is exactly the one described in Proposition 1.

Imagine first that the Incumbent offers a menu of two screening contracts, incurring the cost k. In order to avoid being undercut, the Incumbent must set the repayments as low as possible, namely:[5]

$$D_s^I = \frac{1}{p_s} + \frac{2k}{x_s} \qquad D_r^I = \frac{1}{p_r} + \frac{2k}{x_r}$$

Thus, profits are equal to zero. It can be easily shown that the Incumbent can find an $x_s < 1$ such that screening is possible. Note that the Entrant can undercut the Incumbent by offering one contract only, thereby saving the cost k. Define D_b as the repayment such that:

$$\beta(p_s D_b - 1) + (1-\beta)(p_r D_b - 1) = 0 \quad \Rightarrow \quad D_b = \frac{1}{\beta p_s + (1-\beta)p_r}.$$

Then undercutting is possible if:

$$\frac{1}{p_s} + \frac{2k}{x_s} > \frac{1}{\beta p_s + (1-\beta)p_r},$$

that is satisfied for k large enough. If this happens, the Incumbent goes bankrupt and the Entrant serves the whole market. Even if this condition is not satisfied, a similar strategy can be followed by undercutting only the contract targeting the Risky borrowers. This is done by offering $1/p_r + 2k/x_r - \epsilon$. Also in this case the Incumbent goes bankrupt, and the Entrant serves $1-\beta$ Risky clients, making positive profit. This strategy is always possible. To see why, note that the repayment so that an MFI serving only the Risky borrowers breaks even is $D = 1/p_r$. Then, since $x_r = 1$ at the optimum, we know that $1/p_r + 2k > 1/p_r$. Thus, for any $k > 0$, it is never optimal for the Incumbent to offer two contracts.

Suppose, instead, that the Incumbent offers only one screening contract, as described in the previous section. Then the Entrant can react by offering two contracts: an incentive compatible one as specified in the previous sections, and a second one (also incentive compatible) that undercuts the Incumbent's contract. For instance, if the Incumbent offers a contract targeting the Safe borrowers, than the Entrant can undercut her by setting $x_s = \hat{x}_s + \epsilon$, with $\epsilon > 0$ arbitrarily small. At the same time, she can offer a contract targeting the Risky borrowers, and adapt the informational rent consequently setting $D^E = \hat{D}(\hat{x}_s + \epsilon)$. By doing this, she also earns the profit of the Incumbent, that is forced out of the market.

[5] The conditions are easily computed by solving the following equations for D_s and D_r respectively:

$$x_s \frac{\beta}{2}(p_s D_s - 1) - 2\frac{\beta}{2} p_s k = 0$$
$$x_r \frac{(1-\beta)}{2}(p_r D_r - 1) - 2\frac{(1-\beta)}{2} p_r k = 0$$

This strategy is not viable if $k > \Pi_s^I := \overline{k}$. In this case, in fact, by offering a menu, the Entrant reduces her profit compared to a single screening contract. This is exactly the type of market described in Proposition 1.

To conclude the analysis, note that, for the argument described in the previous section, we know that if the Incumbent chooses a screening strategy, she does so by targeting the Safe borrowers. Thus, whenever $k < \Pi_s^I$ the Incumbent does not set a single incentive compatible contract to induce screening. She does better by offering one contract with $D = D_b$.[6] Then the Entrant's best strategy is to offer one contract as well, setting $D = D_b$.[7] Hence, also in this case, in equilibrium both MFIs offers only one contract.

4. The Altruistic Incumbent (AI Model)

We now turn to consider a different behavioral assumption: we assume that the Incumbent MFI is altruistic. Examining such a case is relevant since microfinance was invented for humanitarian reasons. It has been thought of as a poverty reducing tool, based on the idea that poor people have a relevant - but unexplored - amount of entrepreneurial skills that ought to be utilized: the poor must be helped to help themselves.

Nowadays, microfinance markets are characterized by a heterogeneous lot of institutions, spanning from small humanitarian NGOs to big international financial institutions. An economic theory on microfinance cannot put aside the fact that some important players in the game are not merely profit maximizing.

In this section, we model a situation in which a socially motivated Incumbent is followed by a profit-maximizing Entrant. Our goal is to understand how (and if) the presence of an altruistic firm influences the Entrant's strategy, the borrower welfare and the market equilibrium. Our modeling strategy is based on the observation that, in several markets, the first MFIs to enter the market are not profit-maximizing institutions. Business-oriented MFIs tend to enter the market at a later stage (See, for instance Navajas et al., 2003, Lützenkirchen and Weistroffer, 2012).[8] Of course, we do not claim this to be a regularity of all microfinance markets. But, reassuringly, the results of this section resist also when the model is solved simultaneously.[9] Therefore, we believe our results to also be interesting in markets where this assumption is not valid.

This setup is also interesting from a theoretical point of view, since we model a situation of asymmetric competition between differently motivated institutions.[10]

[6] This is an equilibrium under the assumption that MFIs prefer to stay in the market at zero profit rather than shutting down.

[7] Note that the same reasoning would apply in a simultaneous setup.

[8] We observed directly a similar dynamic in Odisha, India, where we conducted an extensive survey on Self Help Groups. The NGO PRADAN was for a long time the only provider of microfinance. They focused on the creation of SHGs and on linking them with commercial banks. In 2010, other MFIs entered the region (SKS Microfinance, Asmitha Microfin Limited) offering standard micro-loans with a more commercial approach.

[9] A formal proof is available upon request.

[10] A very similar setup has been analyzed in the wider literature on mixed oligopoly, in which public and

There are different possible ways to model altruistic behavior. We consider one interesting instance. We consider a sophisticated form of altruism that we label *Smart Altruism*. This is the behavior of an MFI that also takes into account the effect her strategy has on the Entrant's clients. Therefore, an altruistic MFI maximizes the utilities of all the borrowers in the market. This behavioral assumption fits a market in which the Incumbent is an MFI running a carefully-engineered program.[11]

Note that offering two contracts is always a dominated strategy for the Altruistic Incumbent if $k > 0$. If she offered two contracts, for the same reasoning in Section 3, the Entrant could always undercut the Incumbent and drive her out of the market.

A smart altruistic MFI is concerned with the welfare of her clients *and* with the welfare of the customers served by her competitor. She faces the following maximization problem:

$$\max_{D^I, x^I} x^I B^I(C^I, C^E(C^I))[m - P^I(C^I, C^E(C^I))D^I]+ \\ x^E B^E(C^I, C^E(C^I))[m - P^E(C^E(C^I), C^I)D^E(C^I)] \tag{4}$$

subject to:

$$B^I(C^I, C^E(C^I))x^I[P^I(C^I, C^E(C^I))D^I - 1] \geq 0 \quad NBC$$

The Incumbent has again three options: serve the Safe borrowers (inducing screening), serve the Risky ones (also inducing screening), or target both types. We analyze one by one these options.

Consider first the case in which the Incumbent prefers to serve both types of borrowers. To maximize the borrower welfare the Incumbent sets D^I as low as possible, so that the NBC binds, and x^I as high as possible. We have, therefore, $D^I = D_b := \frac{1}{\beta p_s + (1-\beta)p_r}$ and $x^I = 1$. The Entrant can only set an identical contract, or else she is out of the market.

Consider now the screening strategies. For each of the available options (serving the Risky or the Safe borrowers), we have to show how the Incumbent behaves in order to maximize the total borrower welfare. The results are summarized in the next lemma.

Lemma 3. *If the Incumbent behaves as a Smart Altruistic MFI and wants to induce screening, she optimally sets:*

- $D^I = 1/p_s$ *if she wants to serve the Safe type only,*
- $D^I = 1/p_r$ *if she wants to serve the Risky type only,*

Proof. See 7 □

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private firms compete in the same market. See De Fraja and Delbono (1990) for a survey.

[11] The behavior of a small, NGO-based program would probably be better described by a *naïve* form of altruism, in which the NGO only maximizes her clients' utility subject to a non-bankruptcy constraint. The equilibrium, in such case, is trivial.

This result is intuitive but not trivial. It says that in both screening strategies, the best the Incumbent can do is to set her repayment as low as possible, making her NBC binding. The lemma is important for understanding how an altruistic attitude by the Incumbent can influence the strategic behavior of the profit-maximizing Entrant. There are two components to take into account: how the Incumbent's altruism changes the Entrant's outside options, and the competitive effect of lower prices.

First, consider the case in which the Incumbent serves the Safe borrowers. In this setup the Entrant cannot undercut the Incumbent's contract, so the relevant outside option is simply zero. Lemma 3 shows that the Incumbent's altruism affects the Entrant's profit only insofar as it changes her outside options. The Entrant's contract is otherwise independent of the Incumbent's one. Moreover, the Incumbent in this case faces a trade off: a lower D implies a lower x to attain screening, so that a lower interest corresponds to more rationing. This mechanism makes it less attractive for a Smart Altruistic Incumbent to specialize in the Safe borrowers. To reduce the repayment, she has to ration more than a profit-maximizing firm would do, without inducing a counterbalancing reaction of the Entrant.

Consider now the case in which the Incumbent serves the Risky borrowers. Given our assumptions, the price $1/p_r$ can be larger or smaller than R_s. If $1/p_r < R_s$, the Entrant cannot undercut the Incumbent by setting $D^E = R_s$ as in Section 2.2. So, the Incumbent's altruism changes the outside option, and has a strong competitive effect on the Entrant's strategy. The impact is less important when $1/p_r > R_s$: in that case, the Entrant has the same outside option as in Section 2.2 (i.e. to set $D^E = R_s$), but x^E can now be larger and, as a result, all the borrowers are better off compared to in the profit maximizing model.

Despite the observations above, under the current assumptions a Smart Altruistic Incumbent prefers not to play a screening strategy. The result is described in the next proposition:

Proposition 2. *The model with a Smart Altruistic Incumbent has a unique Subgame Perfect Equilibrium, in which both the Incumbent and the Entrant set* $C^I = C^E = (1, D_b)$.

Proof. See Appendix □

The result follows immediately through the comparison of the total borrower welfare in the three possible situations. It is important to note, though, that the result depends on the assumption that each MFI can serve serve the whole market. When this assumption is relaxed, i.e. if we assume that MFI i faces a capacity constraint $\alpha^i < 1$, with $i = I, E$ and $\alpha^E + \alpha^I \leq 1$, the result changes. To simplify the analysis, assume that $\alpha^E < (1-\alpha^I)(1-\beta)$[12]. The capacity constraints have two effects: on the one hand, they limit the positive impact of the Incumbent's altruism (the Incumbent can serve a smaller number of clients) making the other strategies comparatively more attractive; on the other, they change the Entrant's outside options, since she can always serve the residual demand charging monopolistic prices.

A Smart Altruistic Incumbent with capacity constraint opts for specialization under simple conditions. Note that α^E represents the residual demand the Entrant can serve if all

[12] The result holds also when this assumption is relaxed.

borrowers prefer the contract offered by the Incumbent since, by assumption, $\alpha^E \leq (1-\alpha^I)$. Let $\bar{\alpha}^I$ be the value of α^I for which the Incumbent is indifferent between engaging in a screening strategy and serving Both types. Finally, define:

$$\Pi_{ResR} := \alpha^E(1-\beta)(m-1)$$
$$\Pi_{ResB} := \alpha^E[\beta(m-1)+(1-\beta)(p_r R_s - 1)]$$

Π_{ResR} and Π_{ResB} represent the outside options of the Entrant, that is the profit she can make by serving the residual demand. The equilibrium under our new assumptions is described in the next proposition:

Proposition 3. *Consider the model with a Smart Altruistic Incumbent and the capacity constraints. Then there exists an $\bar{\alpha}^I$ such that:*

- *When $\alpha^I \geq \bar{\alpha}$, in equilibrium the Incumbent sets $C^I = (1, D_b)$ and the Entrant sets $C^E = (1, R_s)$ if $\Pi_{ResR} < \Pi_{ResB}$ and $C^E = (1, R_r)$ otherwise.*
- *When $\alpha^I \leq \bar{\alpha}$, there exist equilibria with screening provided that $\alpha^I \geq (1-\beta)$ [$\alpha^E \geq (1-\beta)$] where the Incumbent serves the Risky [Safe] borrowers.*

Proof. See 7 □

Due to the large number of cases to be examined, the values of the threshold $\bar{\alpha}^I$ and the full characterization of the screening contracts are reported in the Appendix.

In order to make screening possible, it is first of all essential that the market of Risky borrowers be fully covered. Otherwise, the excluded borrowers would sign a contract designed for the Safe ones. Secondly, α^I should not be too large. The Altruistic Incumbent, in this case, faces a trade-off. When engaging in a screening strategy, she can serve a smaller number of clients (β or $1-\beta$). But in a screening strategy she can give a larger rent to her clients since, if she targets both types, the Safe borrowers *de facto* subsidize the Risky ones.

When α^I is relatively small, there are two effects. On the one hand, D^I must (weakly) increase as α^I decreases, since the Entrant's outside option of serving the residual demand becomes more attractive. On the other, since the direct impact on her borrowers' welfare decreases (she can serve a smaller number of them), the Incumbent must rely more on the competitive effect induced by lower prices. The best way to maximize the latter effect is to induce the Entrant to engage in a screening strategy and avoid her behaving as a monopolist on the residual demand.

It is interesting to observe that $\bar{\alpha}$ is decreasing in β. This implies that the riskier the market, the larger the range of parameters for which equilibria with screening exist.

Note that when the altruistic Incumbent serves the Risky borrowers, in equilibrium rationing is bounded to be extremely low ($x_s^E = 1-\epsilon$). In the profit-maximizing Incumbent case, the amount of credit can be much lower since $\hat{x}_s$ can take any value in the interval $[0, 1]$. This is due to the fact that the binding incentive constraint is the one ensuring that the Risky borrowers do not prefer the contract designed for the Safe borrowes. Now, when the Incumbent is altruistic, the Risky borrowers are already given the maximal possible rent, and this mitigates the necessity to ration the Safe ones.

5. Discussion of the main results

The models presented above have a number of relevant implications in terms of policy. In what follows, we briefly discuss three issues: (i) how a price cap would influence borrower welfare; (ii) whether the presence of an altruistic MFI can discourage entry of competitors and limit the outreach of microfinance; (iii) the use of the loan size as an indicator of the outreach of microfinance programmes.

5.1. Price caps

Imposing a price cap can be a sensible policy if MFIs are believed to be able to extract exploitative rent from the borrowers they serve. Our stylized model, shows that MFIs are able to extract rent only when offering a menu of contracts is too costly. We believe this assumption to be realistic for many microfinance institutions.

To understand the consequences that a price cap could bring to our setup, assume that policy makers impose a cap $\overline{D}$ to the repayment, with $\overline{D} \in [R_s, R_r]$. Suppose also that k is large enough and that MFIs play a screening strategy in equilibrium. In such a situation, the price cap can cause a reduction of the profit of the MFI serving the Risky borrowers. If this happens, Risky borrowers can enjoy a larger rent and screening is possible for larger values of x_s. Thus, a price cap, would increase the profit of the MFI serving the Safe borrowers who, in turn, would still get no rents, but would benefit from being less rationed.

Following this analysis, a price cap could also lead to a drastic change of equilibrium if one of the conditions necessary for screening to take place, $k > \Pi_i^s$, is not satisfied after the imposition of the price cap. In this case, both MFIs react to the regulation by offering the same contract, and both make no profit.

Our model predicts a similar phenomenon in the case of an Altruistic Incumbent with capacity constraints. Under these assumptions, a price cap would limit the ability of the Entrant to behave as a monopolist on the residual demand, and reduce the incentives of the Incumbent to induce a screening equilibrium.

In all these scenarios, there is a clear advantage for borrowers, but the price cap could have dramatic effects on the profit of MFIs and could reduce the variety of products within the market. This highlights how important it is, before implementing this type of policy, to investigate the effects of a drop in profits on the outreach and on the incentives to invest in microfinance.

5.2. The effect of altruism and capacity constraints on entry

The presence of an altruistic MFI has the obvious consequence of increasing borrower welfare. However, some have pointed out that it could also hamper the development of a competitive and open financial sector: a strongly socially motivated player could discourage possible investors from entering the market, due to overly-harsh price competition (Rhyne, 1998; Prahalad, 2004; Christen and Drake, 2002).

In contrast to this, our model shows that when capacity constraints limit the outreach, the presence of an altruistic MFI can also have a positive impact on the profit-maximizing Entrant. Consider a situation in which the Incumbent serves the Risky borrowers and the Entrant serves the Safe ones. In the screening equilibria of the Altruistic Incumbent model there are two contrasting effects. On the one hand, the Entrant, serving the Safe borrowers, can reduce rationing to the minimum. This has a positive effect on her profits. On the other, the Incumbent's offer is so low that even the Safe borrowers must be offered a rent. This clearly reduces the profit. For a large range of the parameters, the former effect outweighs the latter, so that the Entrant is better off when the Incumbent is Altruistic. One example is given in Figure 4.

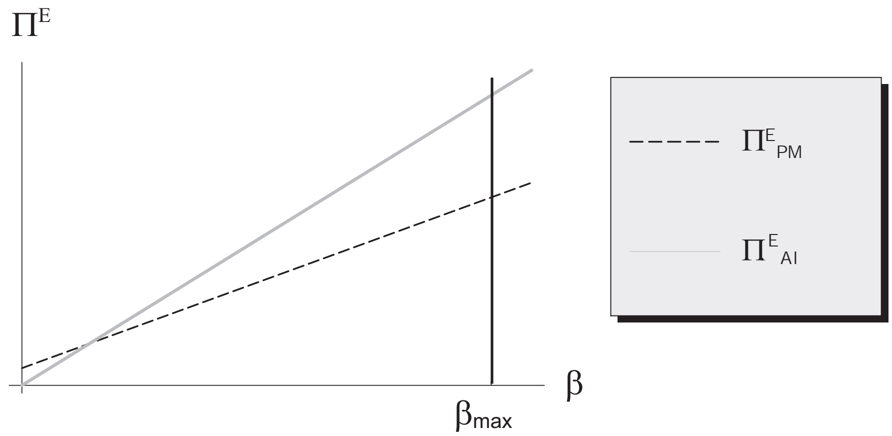

Figure 2: Entrant Profit: Comparison of AI model and PM model

The figure shows the Entrant's profit as a function of β. We considered an example in which $1/p_r > D^I_{min}$. The dashed line Π^E_{PM} represents the Entrant's profit in the Profit-Maximizing Incumbent model, when a screening equilibrium prevails. The grey line labeled as Π^E_{AI} shows instead the Entrant's profit in the Altruistic Incumbent model when she serves the Safe borrowers and the Incumbent serves the Risky ones. Let $\beta_{max} = \bar{\alpha}(\beta)^{-1}$. Then for $\beta < \beta_{max}$ – that is in the interval in which the Altruistic Incumbent prefers to serve the Risky borrowers – Π^E_{AI} is bigger than Π^E_{PM} for β big enough. In other words, the negative effect due to harsh price competitions can be outweighed by the positive effect of less rationing.

In order to get this effect, α must be relatively small and the pool of borrowers must be heterogeneous enough (that is $p_s - p_r$ must be large).[13] The validity of these conditions ought to be tested. But the result seems to be in line with the findings in McIntosh et al. (2005).

[13]By equating Π^E in the two different models, we can solve for the value of β in which the two curves

5.3. Credit Rationing vs Depth of Outreach

A growing number of papers study the evolution of microfinance outreach to the poor. They investigate whether the hunt for profitability and the increase in competition are leading to a mission drift, i.e. a shift towards relatively wealthier market niches. Most of these papers use the loan size as a proxy for outreach (See Cull et al., 2009b, Cull et al., 2007, Mersland and Øystein Strøm, 2010). The logic is that, in order for MFIs to serve poorer clients, they must offer smaller loans. Empirical evidence suggest that this measure is strongly related to other proxies for outreach to the poor, like the fraction of clients living in rural areas or the fraction of women served. This supports the general validity of such a measure.

Yet our model suggests that some caution is necessary when taking this approach. In an oligopolistic market with asymmetric information, the size of the loan can also be used as a tool to select borrowers. Even assuming that borrowers are all equal in terms of wealth, competing MFIs can have incentives to reduce the size of the loan in order to screen borrowers and extract more rent. Thus, any assessment of microfinance outreach based on a measure such as the loan size can be capturing simultaneously two different effects. More research is necessary in this respect.

6. Conclusions

Microfinance has attracted an important variety of actors, pursuing different objectives and competing with each other in order to attract clients. Our stylized model describes the interaction between these actors in a tractable framework, capturing some of the special features of microcredit markets. We explain why MFIs need to standardize their offer at the firm level, and how the cost they face can shape the market equilibrium and the interest rate.

Our results are important for a series of reasons: First, we show that increasing competition does not necessarily lead to an improvement in borrower welfare. On the contrary, competition can facilitate screening when MFIs are unable to offer a menu of contract. This helps them to extract more rent. The conditions predicted by our model for screening to take place are fairly general; nonetheless, we believe that an empirical test of their validity would be an important contribution for practitioners and policy makers.

Second, we show how important it is to take into account the different motives of MFIs. The interaction between competing MFIs leads to remarkably different equilibria when altruism is modeled explicitly.

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intersect, say β^*. Then, by simple algebra, it can be shown that $\beta^* \in [0, \alpha]$ if, and only if:

$$\alpha \leq \frac{p_r(p_r R_s - 1) + p_s}{p_r(m + p_r R_s - 1)}$$

Third, when modeling altruism, our results depend crucially on whether MFIs are financially constrained or not. Despite large investments in the sector, a large share of MFIs still consider the lack of resources as one of the main obstacles to their expansion. Our model provides some indications as to how the market equilibrium can change as the investments (or the subsidies) in the sector increase.

Lastly, our model provides a simplified framework to re-think a number of, extremely relevant themes, such as the imposition of a price cap, the influence of altruism on entry and the measurement of outreach.

Of course, an exhaustive analysis of the effects of competition ought to also consider a series of closely-related issues (for instance multiple borrowing and information sharing between lenders) and other important dimensions defining microfinance contracts (duration, frequency of repayment, lending mechanism). Considering all these elements in a comprehensive setting is difficult also due to a scarcity of adequate data. Our stylized model provides a first approach in this direction, but more research is needed in order to understand the way microfinance institutions compete with one another.

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Appendix

Proof of Lemma 1. Suppose the Incumbent wishes to serve the Safe borrowers only, and that she offers the contract described in Lemma 1. We show that the Entrant's optimal reaction is to offer a screening contract. We first compute the profits the Entrant achieves serving the Risky borrowers only, that is when $B^E(C^I, C^E) = 1 - \beta$. Her maximization problem in this case is:

$$\max_{x^E, D^E} \Pi^E_{rs} = (1-\beta)x^E(p_r D^E - 1)$$

In order to have $B^E(C^I, C^E) = 1 - \beta$, we need the following conditions to hold:

$$\begin{aligned} D^E &\le R_r && PC1 \\ D^I &\le R_s && PC2 \\ x^E p_r(R_r - D^E) &\ge x^I p_r(R_r - D^I) && IC1 \\ x^I p_s(R_s - D^I) &\ge x^E p_s(R_s - D^E) && IC2 \end{aligned}$$

Consider first the constraints $PC1$ and $IC1$. The IC1 is always binding, since the left-hand side is decreasing in D^E. Solving it for D^E we get: $D^E = R_r - \frac{x^I}{x^E}(R_r - D^I)$. What about x^E? Substituting D^E in the profit function we get:

$$\Pi^E_{rs} = (1-\beta)x^E[p_r R_r - p_r \frac{x^I}{x^E}(R_r - D^I) - 1] = (1-\beta)(x^E p_r R_r - x^E - p_r x^I(R_r - D^I))$$

that is clearly maximized for $x^E = 1$ given that $p_r R_r = m > 1$. So the Entrant can set: $x^E = 1$ and $D^E = R_r - \frac{x^I}{x_r}(R_r - D^I)$, that gives her the expected profit:

$$\Pi^E_{rs} = (1-\beta)[(m-1) - p_r x^I(R_r - D^I)]$$

This profit must be compared with the Entrant's outside options. She can target both types of borrowers, undercutting the Incumbent's contract. This can be done by setting $x^E = 1$ and $D^E = D^I$. The profit is then $\Pi_{Both} := \beta(p_s D^I - 1) + (1-\beta)(p_r D^I - 1)$.

The Entrant serves the Risky borrowers when the following condition holds true:

$$(1-\beta)[(m-1) - p_r x^I(R_r - D^I)] \ge \beta(p_s D^I - 1) + (1-\beta)(p_r D^I - 1)$$

Solving the inequality for x^I we find the condition:

$$x^I \le \hat{x}^I_s(D^I) := \frac{(1-\beta)(m-1) - \beta(p_s D^I - 1) + (1-\beta)(p_r D^I - 1)}{(1-\beta)p_r(R_r - D^I)} \tag{.1}$$

Because of our assumptions, Π_{Both} might be negative. In that case, the condition above becomes:

$$(1-\beta)[(m-1) - p_r x^I(R_r - D^I)] \ge 0$$

Solving the inequality for x^I we find the alternative condition:

$$x^I \le \hat{x}^I_s(D^I) := \frac{R_r(m-1)}{m(R_r - D^I)} \tag{.2}$$

Note that, in both cases, $\hat{x}_s^I(D^I)$ is not necessarily in $[0,1)$. If $\hat{x}_s^I(D^I)$ is greater than one, then screening is possible for any $x^I < 1$.

We still have to show that these values of $\hat{x}_s^I(D^I)$ make screening possible. We have to verify that given the optimal reaction of the Entrant, the value $\hat{x}_s$ satisfies also condition (IC2). Replacing $x^E = 1$ and $D^E = R_r - \frac{x^I}{x^E}(R_r - D^I)$ in the IC2 we get:

$$x^I(R_s - D^I) \geq [R_s - R_r + x^I(R_r - D^I)] \Rightarrow x^I(R_s - R_r) \geq R_s - R_r$$

that is satisfied for any $x^I \in [0,1)$. □

Proof of Lemma 2. Suppose the Incumbent wants to specialize in the Risky sector, inducing the Entrant to serve the Safe borrowers. The Entrant solves this maximization problem:

$$\max_{x^E, D^E} \Pi_{sr}^E = \beta x^E(p_s D^E - 1)$$

To have $B^E(C^I, C^E) = \beta$, the following conditions must be fulfilled:

$$\begin{aligned} D^E &\leq R_s && PC1 \\ D^I &\leq R_r && PC2 \\ x^I p_r(R_r - D^I) &\geq x^E p_r(R_r - D^E) && IC1 \\ x^E p_s(R_s - D^E) &\geq x^I p_s(R_s - D^I) && IC2 \end{aligned}$$

We have to consider two possible cases: (i) the Incumbent sets $D^I \geq R_s$; (ii) the Incumbent sets $D^I < R_s$. We show that as long as $D^I \geq R_s$ the Incumbent can raise Π_{sr}^E by setting a lower D^I. But if $D^I < R_s$ the Entrant's profit might decrease because a lower D^E (necessary to have screening) is only partly compensated by a higher x^E.
(i) $D^I \geq R_s$. This is the relevant case when the Incumbent is profit-maximizing. Consider first the IC2. As $D^I \geq R_s$ the RHS is negative, and the PC binds. Thus the Entrant can set $D^E = R_s$. Consider now the IC1. Solving it for x^E we find the condition:

$$x^E \leq \frac{x^I(R_r - D^I)}{R_r - D^E} \tag{.3}$$

that is binding at the optimum. Notice that if $D^I = R_r$, (.3) is true only for $x^E = 0$. So the Incumbent must offer a contract with $D^I < R_r$. The Entrant's expected profits are then:

$$\Pi_{sr}^E = \beta \hat{x}_s^I(D^I)(m-1) \tag{.4}$$

This must be compared with the Entrant's outside options. She can:

1. Target both types by offering a non incentive-compatible contract characterized by $D^E = R_s$ and $x^E = 1$. This strategy gives profit $\Pi_{Both} = \beta(m-1) + (1-\beta)(p_r R_s - 1)$. In this case, for the Incumbent to find it preferable to serve the Safe types, we need $\Pi_{sr}^E \geq \Pi_{Both}$. In formulas:

$$\beta x^E(m-1) \geq \beta(m-1) + (1-\beta)(p_r R_s - 1) \Longrightarrow x^E \geq \left(1 + \frac{(1-\beta)(p_r R_s - 1)}{\beta(m-1)}\right)$$

It is clear that this condition is satisfied only if $(1-\beta)(p_r R_s - 1) < 0$, i.e. there must be some cross-subsidization. Replacing x^E with (.3) we get:

$$D^I \leq R_r - \frac{1}{x^I}\Big[1 + \frac{(1-\beta)(p_r R_s - 1)}{\beta(m-1)}\Big](R_r - R_s)$$

2. Target the Risky sector, undercutting the Incumbent: As shown above, to induce screening the Incumbent must set $D^I = R_r - x^E/x^I(R_r - R_s)$. We can determine the relevant value of x^E by solving the inequality :

$$\begin{aligned} &\beta x^E(m-1) \geq (1-\beta)[(m-1) - p_r x^E(R_r - R_s)] \Longrightarrow \\ &x^E \geq \frac{(1-\beta)(m-1)}{\beta(m-1) + (1-\beta)(m - p_r R_s)}. \end{aligned}$$

Now replacing again x^E with (.3) we get:

$$D^I \leq R_r - \frac{1}{x^I}\Big[\frac{(1-\beta)(m-1)}{\beta(m-1) + (1-\beta)(m - p_r R_s)}\Big](R_r - R_s)$$

Summing up, if we define

$$\tilde{x}_s := \max\left\{1 + \frac{(1-\beta)(p_r R_s - 1)}{\beta(m-1)}, \frac{(1-\beta)(m-1)}{\beta(m-1) + (1-\beta)(m - p_r R_s)}\right\} \tag{.5}$$

then $\tilde{D}^I_r(\tilde{x}_s) = R_r - \frac{1}{x^I}\tilde{x}_s(R_r - R_s)$ gives the upper bound for D^I.
(ii) $D^I < R_s$. This case is relevant when the Incumbent is altruistic. We can rewrite the incentive constraints in this way:

$$\begin{aligned} x^E p_s(R_s - D^E) \geq p_s(R_s - D^I) \quad &\Rightarrow \quad D^E \leq R_s - \frac{x^I}{x^E}(R_s - D^I) \\ p_r(R_r - D^I) \geq x^E p_r(R_r - D^E) \quad &\Rightarrow \quad D^E \geq R_r - \frac{x^I}{x^E}(R_r - D^I) \end{aligned}$$

The equations above delimit an interval of contracts satisfying both incentive constraints. For $x^E < 1$ this interval for D^E exists and has a strictly positive measure. So for any contract offered by the Incumbent with $D^I < R_s$ the Entrant can make screening possible by choosing $x^E < 1$ and $D^E < D^I$, making the Safe borrower's incentive constraints binding. By doing this, she can earn up to $\Pi_{sr} \cong x^E\beta(p_s D^I - 1)$. She chooses this strategy if, and only if, it gives her a higher profit than the possible outside options: serving the residual demand or undercutting the Incumbent's contract. Let then D^I_{min} be the minimal value of D^I making the Entrant indifferent between the screening profit and the outside option. That gives the lower bound for D^I.

□

Proof of Proposition 1. (i) We know that when the Incumbent offers a contract like the one described in Lemma 1, the Entrant reacts by offering a contract that makes screening possible. Thus the Incumbent profits are given by $\Pi^I_s = \beta\hat{x}_s(R_s)(m-1)$. Consider Π^I_s as a function of β. It is easy to show that Π^I_s is positive and concave, first increasing and then decreasing, for $\beta \in [0, \beta_0]$, where $\beta_0 = \frac{m(R_r - R_s)}{m(2R_r - R_s) - R_r}$. It is equal to zero at the point $\beta = \beta_0$.

We need to compare Π_s^I to Π_r^I. We know that, if the Incumbent wants to serve the risky borrowers, the outside options of the Entrant (serving both types and undercutting the Incumbent serving the Risky borrowers) are equal in the point:

$$\hat{\beta} = \frac{\sqrt{(m-1)^2 m R_r^2 (R_r - R_s)(R_r - mR_s)} + m(R_r - R_s)(mR_s - R_r)}{(1 + (m-3)m)R_r^2 + m(1+m)R_r R_s - m^2 R_s^2}$$

Now, consider the interval $\beta \in [0, \hat{\beta}]$ and suppose the Incumbent prefers serving the Risky borrowers. Here the best outside option for the Entrant is to serve the Risky borrowers, undercutting the Incumbent. Note that, by Lemma 2, it must be that in this interval $\Pi_r^I = \Pi_s^E$. In fact a profit-maximizing incumbent always chooses the highest possible D^I making the Entrant indifferent between the screening strategy and the outside option. Now, by contradiction, suppose that $\Pi_s^I < \Pi_r^I$. By the equality before, that would imply that $\Pi_s^I < \Pi_s^E$. Simple algebra shows that $\hat{x}_s^I(R_s) \geq \tilde{x}_s^E$ (compare equations .1 and .2 to equation .5), i.e. that the level of rationing chosen by the Incumbent when serving the Safe borrowers is smaller the the level of rationing that the Entrant would choose when serving the Safe borrowers. Hence it must be that $\Pi_s^I \geq \Pi_s^E$.

Consider now the interval $\beta \in [\hat{\beta}, 1]$. Π_r^I is a piecewise function, and in the interval we are considering it is strictly convex and decreasing in β. It crosses Π_s^I twice in the points $\beta = \hat{\beta}$ and $\beta = \beta_0$, where it becomes negative (see Figure 1). Given the properties of Π_s^I, it follows that $\Pi_s^I > \Pi_r^I$ in the interval $[\hat{\beta}, 1]$.

(ii) When $\beta > \beta_0$, both Π_s^I and Π_r^I are negative. Therefore the Incumbent sets $x^I = 1$ and D^I as low as possible to avoid being undercut, namely $D^I = 1/p_b$. The Entrant can only replicate this contract and both MFIs make zero profit. □

Proof of Lemma 3. Suppose first that the smart altruistic Incumbent wants to serve only the Safe sector, and that she wants to induce the Entrant to engage in a screening strategy. As shown in Lemma 1 this is done by offering $x^I \leq \hat{x}_s^I(D^I)$. We have to consider the effects of her choice on the Safe borrowers she serves *and* on the Risky borrowers the Entrant serves.

We show, first of all, that when $x^I = \hat{x}_s^I(D^I)$, the Entrant's optimal contract does not depend on the value of D^I. We know that the Entrant's reaction is to offer $D^E = R_r - \frac{x^I}{x^E}(R_r - D^I)$. Substituting for the adequate value of $\hat{x}_s^I(D^I)$, it is very easy to check that the value D^E is independent of D^I. So, what matters is the utility enjoyed by the Safe borrowers. Note that, in all the cases analyzed in Lemma 1, $\hat{x}_s(\cdot)$ is increasing in D^I. So, for an altruistic MFI, there is a tradeoff between offering the borrowers a cheaper contract and rationing them more. To find the optimal solution, we need to substitute for $\hat{x}_s$ in the objective function, that in this case reduces to $\beta x^I p_s(R_s - D^I)$. In the relevant interval, this equation is decreasing and concave in D^I. The NBC reduces to $\beta x^I(p_s D^I - 1) \geq 0$. The MFI chooses the lowest possible value of D^I, that is the value that makes her profit equal to zero. This is given by $D^I = 1/p_s$.

Suppose now that the Incumbent chooses to serve the Risky sector. To maximize the Risky borrower's utility, the Incumbent wants to set x^I as high as possible, namely equal to one, and D^I as low as possible. The value of D^I that makes the NBC binding is $1/p_r$. Note that $1/p_r$ can be smaller or bigger than R_s. As described in Lemma 2, the Incumbent can induce screening by setting $x^I = 1$ and $D^I = 1/p_r$. This endows the borrowers served by the Incumbent with the highest possible rent. At the same time, it has a positive influence on the borrowers served by the Entrant, since tougher price competition forces her to reduce the repayment D^E and increase the value of x^E.

□

Proof of Proposition 2. When the Incumbent serves both types of borrowers, she sets $D = D_b := \frac{1}{\beta p_s + (1-\beta)p_r}$. The total borrower welfare is then:

$$W_b := \beta p_s \left(R_s - D_b\right) + (1-\beta)p_r \left(Rr - D_b\right) = m - 1$$

Note that since $p_r D_b$ might be smaller than one, this strategy can entail cross-subsidization. Suppose now that the Incumbent serves the Safe borrowers. Then, as showed in Lemma 3, the Incumbent sets Ds=1/ps, and the Entrant outside option is simply zero. It follows that $\hat{x}_s^I$ must be such that:

$$(1-\beta)\left((m-1) - p_r \hat{x}_s^I (R_r - 1/p_s)\right) > 0 \quad \Rightarrow \quad \hat{x}_s^I = \frac{(m-1)R_r}{mR_r - R_s}$$

The total borrower welfare is then:

$$W_s := \beta \hat{x}_s^I (m-1) + (1-\beta)p_r \left(R_r - (R_r - \hat{x}_s^I (R_r - 1/p_s))\right) = m - 1 - \frac{(m-1)(R_r - R_s)\beta}{mR_r - R_s}$$

Since the last addendum is always positive, it follows that $W_b > W_s$.
Suppose now that the Incumbent serves the Risky borrowers. Two cases must be considered.
(i) If $1/p_r < R_s$, the Entrant cannot set D_s at his highest, or else the Safe borrowers would also take the Incumbent's contract. In order to make screening possible, the following constraints needs to be satisfied:

$$\begin{aligned} x_s^E p_s (R_s - D_s^E) \geq x_r^I p_s (R_s - D_r^I) \Rightarrow x_s^E (R_s - D_s^E) \geq (R_s - 1/p_r) \\ x_r^I p_r (R_r - D_r^I) \geq x_s^E p_r (R_r - D_s^E) \Rightarrow (R_r - 1/p_r) \geq x_s^E (R_r - D_s^E) \end{aligned}$$

By setting $x_s = 1 - \epsilon$ and $D_s = 1/p_r - \epsilon$, the constraints are simultaneously satisfied and screening is possible. Thus, the total borrower welfare is given by:

$$W_r := \beta p_s (R_s - 1/p_r) + (1-\beta) p_r (R_r - 1/p_r) = m - 1 - \beta (R_r/R_s - 1)$$

It follows that $W_b > W_r$.
(ii) If $1/p_r > R_s$, than the Entrant can set $D_s = R_s$, so that all Safe borrowers enjoy no rent. It follows that the total borrower welfare is given by:

$$W_r := (1-\beta) p_r (R_r - 1/p_r) = (1-\beta)(m-1)$$

Also in this case, $W_b > W_r$. □

Proof of Proposition 3. Suppose first that the Incumbent wants to serve both types of borrowers setting $D = D_b := \frac{1}{\beta p_s + (1-\beta)p_r}$. Then, the Entrant's best response is to target the residual demand. The total borrower welfare is:

$$W_b := \begin{cases} \alpha^I \left(\beta p_s \left(R_s - D_b\right) + (1-\beta)p_r \left(Rr - D_b\right)\right) = \alpha^I (m-1) & \text{if} \quad \Pi_{ResR} > \Pi_{ResB} \\ \alpha^I (m-1) + \alpha (1-\beta) p_r (R_r - R_s) & \text{if} \quad \Pi_{ResB} > \Pi_{ResR} \end{cases}$$

W_b is an increasing, linear function of α^I.

Consider now the case in which the Incumbent wants to serve the Safe types only. She can set $D_s = 1/p_s$, and the Entrant's outside option is to serve the residual demand. Following the same procedure explained in Lemma 1, to induce screening the Incumbent must set x^I such that the following conditions are satisfied:

$$(1-\beta)[(m-1) - p_r x^I (R_r - 1/p_s)] \geq \Pi_{ResR} \quad \text{if} \quad \Pi_{ResR} > \Pi_{ResB}$$
$$(1-\beta)[(m-1) - p_r x^I (R_r - 1/p_s)] \geq \Pi_{ResB} \quad \text{if} \quad \Pi_{ResB} > \Pi_{ResR}$$

Let $\hat{x}_s$ be the value of x^I such that the the relevant condition above is satisfied with strict equality. Note that $\hat{x}_s$ is a function of α^E. Then the total borrower welfare is given by:

$$W_s := \beta \hat{x}_s (m-1) + (1-\beta) p_r (R_r - (R_r - \hat{x}_s (R_r - 1/p_s))$$

Note that this function does not depend on α^I and is always positive.

Suppose now that the Incumbent prefers to target the Risky borrowers only. The Incumbent would like to set D^I as low as possible, namely $D^I = 1/pr$ and $x^I = 1$. But $1/p_r$ might be smaller than R_s, and therefore the Incumbent contract could be affordable also to Safe borrowers. Moreover, if $D^I < R_s$ and the Entrant wants to target the Risky borrowers, the Entrant's profit decreases as D^I decrease. Let $D_{min} < R_s$ be the value of D^I such that the Entrant is indifferent between serving the Safe borrowers only and the relevant outside option. We need to distinguish three cases.

Case 1: $1/p_r > R_s$. The Entrant's best outside option is to set $D^E = R_s$ and undercut the Incumbent. Following the same procedure explained in Lemma 2, to induce screening the Incumbent must set D^I such that the following conditions are satisfied:

$$\beta x_s (D^I)(p_s R_s - 1) \geq \Pi_{ResB}$$

Let $\hat{x}_s(1/p_r)$ be the value of x^I such that the the condition above is satisfied with strict equality. Note, again, that $\hat{x}_s$ is a function of α^E.The total borrower welfare is:

$$W_r := \beta \hat{x}_s (Rs - Rs) + (1-\beta) p_r (R_r - 1/p_r) = (1-\beta)(m-1)$$

that is always positive and independent of α^I.

Case 2: $1/p_r < D_{min} \leq R_s$. The Entrant's outside option is serving the residual demand. Following the same procedure explained in Lemma 2, to induce screening the Incumbent must set D^I as low as possible, namely $D^I = D_{min}$, so that the following conditions are satisfied with strict inequality:

$$\beta x_s (D^I)(p_s R_s - 1) \geq \Pi_{ResR} \quad \text{if} \quad \Pi_{ResR} > \Pi_{ResB}$$
$$\beta x_s (D^I)(p_s R_s - 1) \geq \Pi_{ResB} \quad \text{if} \quad \Pi_{ResB} > \Pi_{ResR}$$

as explained in Proposition 2, x^E can be set arbitrarily close to 1, so that $\hat{x}_s = 1 - \epsilon$, with $\epsilon \in \mathbb{R}^+$. The total borrower welfare is:

$$W_r := \beta \hat{x}_s (Rs - (D_{min} - \epsilon)) + (1-\beta) p_r (R_r - D_{min})$$

that is always positive and independent of α^I.

Case 3: $D_{min} \leq 1/p_r \leq R_s$. In this case, the Incumbent can set $D^I = 1/p_r$ and $x^I = 1$. The Entrant reacts by setting $D^E = 1/p_r - \epsilon$ and $x^E = 1 - \epsilon$, making screening possible. The total borrower welfare is:

$$W_r := \beta(R_s - 1/p_r) + (1-\beta)(R_r - 1/p_r) = (m-1) - \beta\left(\frac{p_s}{p_r} - 1\right)$$

that is always positive and independent of α^I.

We can now compare W_b, W_s, and W_r. First of all, note that screening is possible only if the capacity of the MFI serving the Risky borrower is large enough to serve them all. This is because Risky borrowers can always afford the contract designed for the Safe ones and they always prefer it to being excluded from credit. Second, note that W_b is linear and increasing in α^I, whereas W_s and W_r are independent of α^I but decreasing in α^E. It follows that, when all Risky borrowers can be served, there exists an $\hat{\alpha}$ such that for any $\alpha^I \leq \hat{\alpha}$, $\max\{W_s, W_r\} \geq W_b$. $\square$